Applications of Categorical Mereology
to Metaphysics and Theoretical Biology

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1   ABSTRACT OF PROGRESS

Sometimes a well-developed area of pure mathematical research is revitalized by introducing category-theoretic methods, as for example, to algebraic geometry (the theory of rational and algebraic equations). Or, entirely new areas of pure mathematical research are created by developing new category-theoretic ideas, as for example, in topos theory (the theory of variable sets). Beyond pure mathematics, category-theoretic ideas are applied to explain underlying algebraic patterns, as for example in computer science (the theory of computation).

In all applications of category theory, new results may take different forms. A well-known abstract theorem of category theory may be interpreted in a particular context and thus provide fresh insight. But, more commonly, a genuinely interesting result of applied category theory is not so much an interpretation of a theorem, but the upshot of the *category-theoretic method of thinking.* This means clarification of ideas in some research realm by identifying what is "natural" or "universal." This is the turn towards defining peviously unsuspected categories, functors, natural transformations, (co)limits, adjunctions, sheaves, and so on, in the ambient subject-matter.

A substantial part of the appeal of the category-theoretic method of thinking is that it offers a precise diagrammatic representation of structures and equations. Marking the line between what can be said precisely, and the rest, sharpens the questions that need to be addressed, which is a good thing.

Based on a well-known abstract theorem about adjoint functors, mathematicians have created an elegant category-theoretic method of thinking about *constraints of behavior among the parts of a whole*, which herein is called *categorical mereology.* This article applies that method to selected topics in *metaphysics* and *theoretical biology.* To reach out beyond the mathematical community, extensive references to relevant science are provided.

1.1   Categorical Mereology

Mereology is the study of the relationships between wholes and their parts, and between the parts of a whole (Nagel [1963]). If the wholes are "operating structures" of some sort, that means at least that they change somehow as time goes on: they have some sort of *behavior.* How can the behavior of one part possibly be *compatible* with the behavior of another? How can the behavior of one part possibly *constrain* the behavior of another part? Category-theoretic replies to such questions are published as "behavioral mereology" (Fong et al. [2018])[1].

One way to define the natural numbers within the theory of sets is to start with $0 := \varnothing$, $1 := \{0\}$, $2 := \{0,1\}$ and so on by recursion with n defined to be the set of its predecessors 0 to $n-1$. Already 2 is a useful number. For any subset $U \subseteq X$ the function $X \xrightarrow{\chi_U^X} 2$ defined by $\chi_U^X(x) := 1$ if $x \in U$ else 0 is called the *characteristic function* of U relative to X. The set of all subsets of a set X is denoted by $\mathcal{P} X$. Every function from X to 2 is the characteristic function of some subset of X. For good reasons to do

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[1] https://golem.ph.utexas.edu/category/2019/06/behavioral_mereology.html.

with the algebraic behavior of sets with regard to operations such as (Cartesian) product and union of sets, and by analogy with the powers (exponentiation) of numbers, the set of functions from a set X to a set Y is denoted by Y^X. In particular, the set of characteristic functions of subsets of X is 2^X, for which there is a bijection with $\mathcal{P}\,X$. The categorist says, "2 is a subobject classifier in the category of sets and functions."

The set 2^X is richly endowed with algebraic structure, summarized by saying it is a boolean algebra (Sikorski [1969]), which is "the algebraic correlate of the classical propositional calculus" (Mac Lane and Moerdijk [1992])[2]. 2^X is also a partially ordered set with respect to the subset relation. The categorist says, "a partially ordered set is a category such that each set of morphisms has at most one element." For a function $X \xrightarrow{f} Y$ the induced functions $2^X \xrightarrow{f^{\rightarrow}} 2^Y$ and $2^Y \xrightarrow{f^{\leftarrow}} 2^X$ are functors between these categories. A *basic result* is

Theorem 1. *If $X \xrightarrow{f} Y$ is a function then there exists an adjoint triple*

$$\exists^f \;\dashv\; f^{\leftarrow} \;\dashv\; \forall^f$$

of power set functors:

For $U \subseteq X$ the definitions of the new functors are

$$\exists^f U := \{\, y \in Y \mid (\exists x)(f(x) = y \quad \wedge \quad x \in U)\,\} = \{\, y \in Y \mid f^{\leftarrow}\{y\} \cap U \neq \varnothing \,\} \tag{1}$$

$$\forall^f U := \{\, y \in Y \mid (\forall x)(f(x) = y \implies x \in U)\,\} = \{\, y \in Y \mid f^{\leftarrow}\{y\} \subseteq U \,\} \tag{2}$$

and verification of the adjunctions "as stated, is immediate" (Mac Lane and Moerdijk [1992], p.58). In the special case that f is a projection $X \times Y \xrightarrow{\pi.Y} Y$, for which one considers a subset $U \subseteq X \times Y$ to be the extension of a two-place predicate, the adjoint triple

$$\exists^{\pi.Y} \;\dashv\; \pi.Y^{\leftarrow} \;\dashv\; \forall^{\pi.Y}$$

is the starting point of F. W. Lawvere's invention of categorical logic (Marquis and Reyes [2012]). In the special case that f is a surjection, it is a whole-part relation and the *basic result* is the starting point of "categorical mereology." In (Fong et al. [2018])[3] the first example of "systems considered in terms of their behavior types" is the whole system with parts the pedals and the wheels of a bicycle, See Fig. (1). A surjection $f : X \to Y$ in this context assigns to a possible-behavior x of the whole its corresponding possible-behavor $f(x)$ of the part. The equation $f(x) = f(x')$ for distinct possible-behaviors x, x' of the whole means that they have the same effect, the same corresponding possible-behavior of the part.

In Fig. (2) the independence, compatibility, and constraint of behaviors is illustrated with a diagrammatic representation of surjections.

1.2 Metaphysics

The question "What is a thing?" recurs (Heidegger [1967]). At first one may say a thing just occupies a region of space for some interval time. That is, a thing is a physical thing. More broadly, a thing is that

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[2]It is not necessary that every subset be considered the extension of a proposition (Rescher [1959]). An alternative to (Zermelo-Frankel axiomatic) set theory (with the Axiom of Choice) only considers subsets defined by specified formulas to represent propositions (Devlin [1977]).

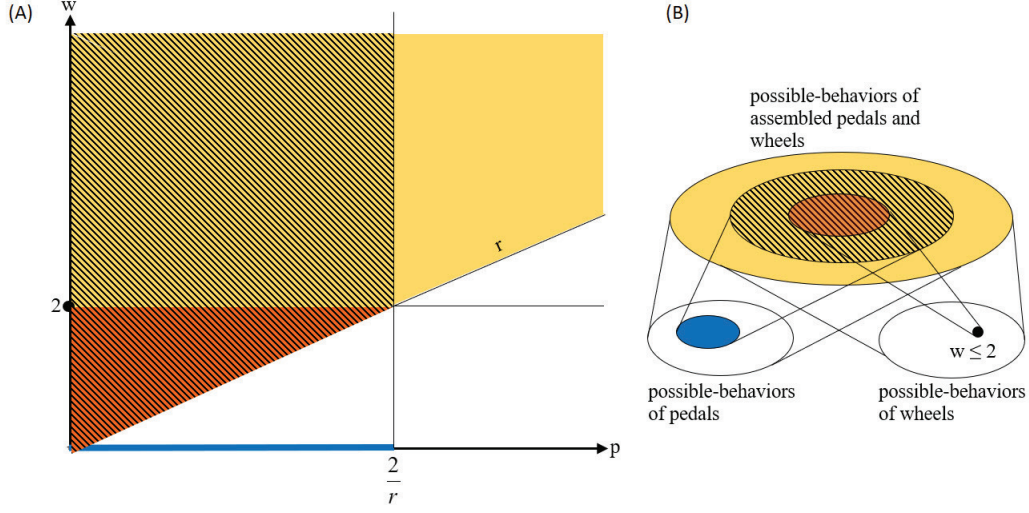[3]See also `https://golem.ph.utexas.edu/category/2019/06/behavioral_mereology.html`.

Figure 1: Wheel speed w and pedal speed p satisfy the constraint relation $w \geq r \cdot p$. (A) The "speed space" of the system (Fong et al. [2018], p.11). (B) Surjections of possible-behaviors.
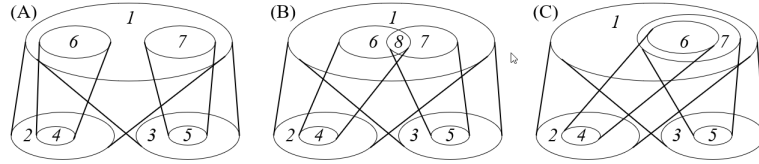

Figure 2: (A) Whole thing 1 with parts 2 and 3 satisfying local constraints 4 and 5, respectively, satisfy completely independent global constraints 6 and 7. (B) As in (A), except there exists a compatible global constraint 8 with respect to 4 and 5. (C) As in (B), except that *every* global behavior in 6 of 1 that is consistent with a local behavior in 4 of 2 *also* satisfies local constraint 5 of 3: 4 "ensures" 5.

which can be thought about: "mental entities, essences, forms, kinds, properties, relations, modes tropes, events, processes, forces, laws, states of affairs, facts, propositions, moments, points, collections, sets, numbers, holes, privations" and so on (Fiocco [2019]). But having any thought at all means behavior of a brain, which is a physical thing. So, what is a physical thing?

The partial-order category of bounded open-intervals of time is denoted by $\mathbb{I}$. For each object $I \in \mathbb{I}$, the assignment to I of the set of sets of bounded open intervals whose union equals I, that is, the set of *covers* of I, is a Grothendieck topology for $\mathbb{I}$. Assume that the set of all things is a "variable set" (Barr et al. [1986]) $\mathcal{X}$, that is, a sheaf $\mathcal{X} : \mathbb{I}^{op} \to \mathcal{S}et$ (Mac Lane and Moerdijk [1992]). Likewise, assume that all possible-behaviors is a variable set, $\mathcal{B} : \mathbb{I}^{op} \to \mathcal{S}et$. The basic metaphysical axiom is that there exists a natural transformation of set-valued sheaves

$$
\begin{array}{ccc}
 & \mathcal{B} & \\
\mathbb{I}^{op} & \Big\downarrow{\scriptstyle\eta} & \mathcal{S}et \\
 & \mathcal{X} &
\end{array}
\tag{3}
$$

3

in which for every bounded open interval I there exists a surjection $\eta_I : \mathcal{B}_I \to \mathcal{X}_I$ assigning to each thing $x \in \mathcal{X}_I$ the set $\eta_I^{-1}(x)$ of possible-behaviors of x, naturally with respect to I. In a sense, a thing *is* its set of possible-behaviors. Thus, for $I \subseteq I'$ there is a commutative diagram

$$
\begin{array}{ccc}
\mathcal{B}_I & \longleftarrow & \mathcal{B}_{I'} \\
{\scriptstyle \eta_I} \downarrow & & \downarrow {\scriptstyle \eta_{I'}} \\
\mathcal{X}_I & \longleftarrow & \mathcal{X}_{I'}
\end{array}
\tag{4}
$$

A basic constraint on the possible-behaviors of things is that "no two things occupy the same region of space at the same time." The equations of Newtonian mechanics, of collision theory, of classical field theory, and so on, are expressible in terms of constraints on the possible-behaviors of things upon one another. A classical "physical reality" must consist of things conforming to *all* such constraints. If there exists a physical reality, then its things must be a consistent choice among all possible-behaviors. But that discussion leaves the Department of Metaphysics down the hall to The Department of Physics.

1.3   Physical Thing

For a model of time choose the continuum $\mathbb{R}$ of real numbers (complete ordered field, (Davis [2005])). Space $\mathbb{E}$ is modelled by a three-dimensional affine space, with its possible choices of origin for coordinate systems, and its group of translations (Whitney [1957]). For any choice of line through a choice of origin there is also the group of rotations. Two subsets of $\mathbb{E}$ are by definition *congruent* if there are choices of rotation and translation that map one upon the other. Hence, congruent sets have the same orientation.

The variety of possible shapes of physical things is "greater than" the variety of the usual topological spaces. An idea of "qualitative shape" may be explained as follows. A *core* is a finite set of *joints*, together with a set of unordered pairs of distinct joints, together with a set of unordered triples of distinct joints. Such a pair is called a *strut*, and such a triple is called a *plate* of the core. Any of these sets may be empty, but the three two-distinct-element subsets of a plate must be struts. There exists a category $\mathcal{C}ore$ of cores as objects and strut-and-plate preserving maps of joints as morphisms. Every morphism is a composition of a surjection followed by an injection of joints. A surjection of a core C upon a core D is called an *analysis* of D by C. It "expands" joints of D as more "detailed" cores contained in C. The motivation for the definition of core is the concept of biological macromolecule (Penner [2016]), but is more generally applicable as an approximation to *any* physical structure.

A joint of a core is isolated if it is not in a strut. A core is *connected* if for any two distinct joints there is a path of struts from one to the other. Cores may be *cleaved* by removal of struts, and *joined* by addition of struts. These operations endow $\mathcal{C}ore$ with additional algebraic structure that will be taken for granted.

For any strut $\{x, y\}$ the *open segment* between x and y is the set $(x, y) \stackrel{dfn}{=\!=} \{\lambda_1 x + \lambda_2 y \mid 0 < \lambda_1, \lambda_2 < 1 \text{ and } \lambda_1 + \lambda_2 = 1\}$ (note bold parentheses). Similary, for any plate $\{x, y, z\}$ the *open triangle* across x, y, z is $(x, y, z) \stackrel{dfn}{=\!=} \{\lambda_1 x + \lambda_2 y + \lambda_3 z \mid 0 < \lambda_1, \lambda_2, \lambda_3 < 1 \text{ and } \lambda_1 + \lambda_2 + \lambda_3 = 1\}$. Open segments and open triangles are topological spaces, and a core determines an adjunction space (Lundell and Weingram [1969]) called the *space* of the core, and the space of a core C is denoted by $\|C\|$. Thus, $\|\_\|$ is a functor $\mathcal{C}ore \to \mathcal{T}op$. Isomorphic cores have homeomorphic spaces.

A *physical thing* with core C is a map $T : \|C\| \to \mathbb{E}$ such that the open segments of struts and open triangles of plates are mapped to open segments and open triangles in $\mathbb{E}$. It is required of such a map that *images of struts and plates never intersect*. In other words, T is an embedding. The image $T(\|C\|) \subset \mathbb{E}$, is called the *realization* of T. For an example, see Fig. (3).
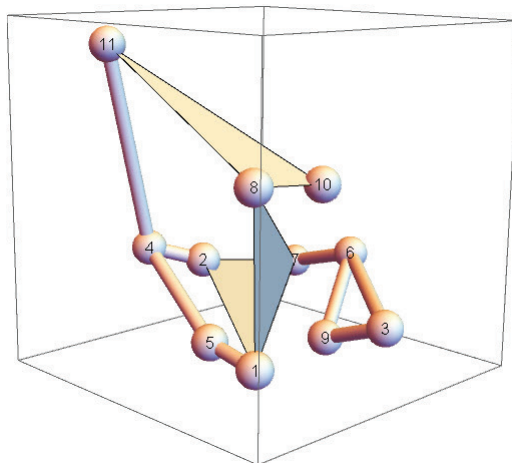
Figure 3: Realization of a physical thing with 11 joints.

In a realization of a physical thing there is a length of each strut, there is an angle between any two distinct struts at a joint, and there is an angle between any two distinct plates joined along a common strut. The entire system of strut lengths, strut angles, and plate angles is called the *conformation* of the realization. The congruence class of a (realization of a) physical thing is called its *physical shape*, and any two congruent physical things have identical conformations. Congruence classes of physical things correspond exactly to pairs consisting of a core and a conformation.

For a bounded open-interval $I \stackrel{abb}{=\!=\!=} (a, b) \subset \mathbb{R}$ of time, a *possible-behavior* of a physical thing T with core C is a map $B : I \times \|C\| \to \mathbb{E}$ such that at each time $t \in I$, $T(t, \_\_) : \|C\| \to \mathbb{E}$ is a physical thing with core C.

Physical things, also known as *objects*, are composed of *amounts of substances* (see below). Possible-behaviors of "variable objects" are called *processes*, and there are two kinds of process: *transportation* and *transformation* of *substance*. All processes are caused by *potential differences* (gravitational, electrical, magnetic, chemical, and so on) and qualified by constraints. Objects are wholes with parts, and some objects may be dis-assembled and re-assembled from their parts. These kinds of objects, therefore, may exist during distinct intervals of time. However, there is a kind of object called a *living being* that exists during exactly one interval of time, its *lifetime* (Beckman and Ames [1998])(Holliday [2010]).

1.4 Theoretical Biology

Coordination of constraints is exquisitely crucial to biology (Hooker [2013]). The "Modern Synthesis" of Darwinian natural selection and Mendelian genetics, updated by the "Central Dogma" of molecular biology, is on the way to synthesis with systems biology. "What Waddington called developmental constraints and epigenetics can now be identified as the layers of molecular regulatory networks and cell-cell communication networks – a web of interactions through which genomic information must percolate to produce the macroscopic phenotype" (Huang [2012]). A proposed universal definition of life emphasizes the *autonomy* of organisms: life is "a complex collective network made out of self-reproducing autonomous agents whose basic organization is instructed by material records generated through the evolutionary-historical process of that collective network" (Ruiz-Mirazo et al. [2004]). The concept of *biological autonomy* has a distinguished pedigree, of which a venerated ancestor is the concept of *autopoiesis*. "An autopoietic machine continuously generates and specifieds its own organization through its operation as a system of production of its own components, and does thsi in an endless turnover of components under conditions of continuous perturbations and compensation of perturbations" (Maturana and Varela [1980]). This concept has percolated from biology to philosophy of science (Meincke [2019]) and to artificial life research (Nomura [2002]). Narrowing

the focus from defining life in general to defining the concept of *organism*, Robert Rosen, in an application of category theory, proposed answering the question, "What is life?" with a definition: "a material system is an organism if, and only if, it is closed to efficient causation" (Rosen [1991]). (A more ambitious application of category theory to the emergence of life and consciousness is in (Baianu et al. [2007]).) A research program on theoretical principles for biology that moves away from gene-centric reductionism towards a holistic, process-centric ontology leads to defining biological organization in terms of *closure of constraints* (Rosslenbroich [2014]) (Moreno and Mossio [2015])(Mossio et al. [2016]). The simplest abstract example in Fig. (4) is explicated in Fig. (5)
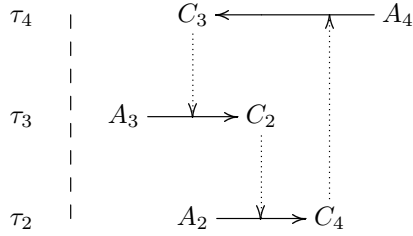

Figure 4: The simplest abstract example of "closure of constraints." Each constraint $C2$, $C3$, and $C4$ influences a process, and is created by a process.
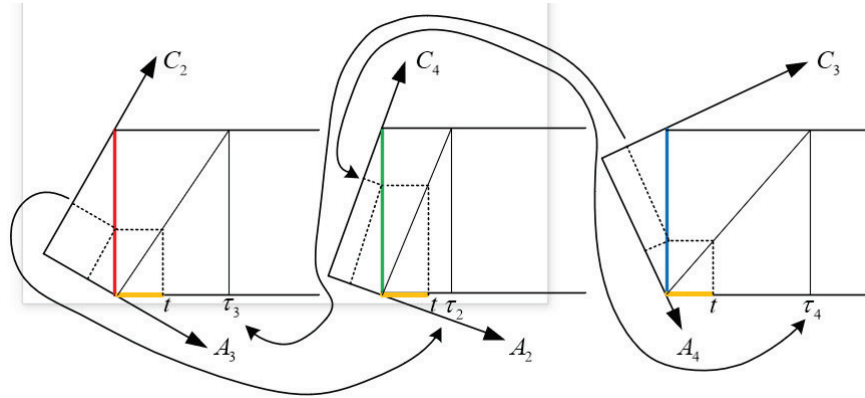

Figure 5: The process $A3 \to C2$ during $[0, \tau_3]$ at time t results in a value of $C2$ that constraints τ_2. At that time t, also process $A2 \to C4$ results in a value of $C4$ that constrains $\tau4$. Closure of the cycle of constraints at time t is process $A4 \to C3$ that constraints τ_3.

There exists exactly one $\mathcal{L}ife$ on the planet Earth (Cleland [2013]), and this whole thing has had, and still has, numerous parts. all organisms are composed of one or more biological cells. All biological cells are containers of substances undergoing processes.

1.5   Container

A *toroidal core* is a connected core such that every strut is a boundary-strut of exactly two plates. By the classification theorem for two-dimensional surfaces (Stillwell [1980], p.69), and the fact that realizations of things disallow self-intersections, any realization of a toroidal core is homeomorphic to a torus of genus n for some integer $n \geq 0$. A generalization of the Brouwer-Jordan Separation theorem Greenberg [1967], p.81), which applies to genus 0, i.e., to spherical core spaces, would state that the realization of a toroidal core

space separates $\mathbb{E}$ into two components, one of which is bounded and the other is unbounded, both having the realization as their common frontier. The bounded component is called the *inside* of the realization, the unbounded component its *outside*. A *container* is a physical thing with a toroidal core.

A *biological cell C* is a container with genus 0. See Fig. (6).
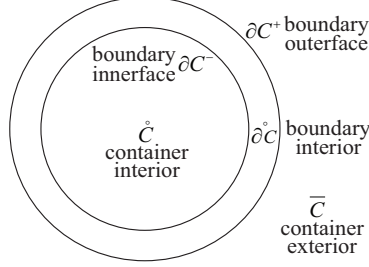

Figure 6: Diagram, notation, and nomenclature for expressing mental model of a container.

Cell boundary $\partial \overset{\circ}{C}$ is a shape between an outer spherical space and an inner spherical space. Cell interior includes other containers such as the nucleus, mitochondria, endoplasmic reticulum (which has a large genus), golgi apparatus, mitochondrion, secretory granules, and so on (Arthur C. Guyton and John E. Hall [2006], p. 13). There are many ways exterior and interior of a cell influence one another via the cell membrane (Pollet et al. [2018]) (Nelson and Cox [2005] pp. 27-39, 390, 414, 1069): translocation (Trowitzsch and Tampé [2018]), exocytosis (neurotransmitter release into synaptic clefts during brain operation) (Takei et al. [1996]), endocytosis, (pinocytosis, phagocytosis) (Arthur C. Guyton and John E. Hall [2006])(See also Isaeva et al. [2014]).

"[C]compartmentalisation constitutes a crucial requirement for the emergence of a form of minimal organisational closure that, in turn, can be a relevant step for the further increase of organised complexity" (Moreno and Mossio [2015]). "Seclusion from the environment was established because of the boundary, within which high concentrations of organic molecules could be accumulated so that an osmotic gradient toward the outer environment was established. A micromilieu was built and defended against environmental influences, which can be regulated and in which special conditions for the reactants of the metabolic systems are maintained. The cell membrane establishes a relatively closed compartment in which metabolism and genetic information are internalized and protected against destructive influences" (Rosslenbroich [2014]). If you suppose some words (e.g., "spirit") are dead metaphors with ancient literal roots (e.g., "breath"), then analogously, the metaphorical "view from within" in consciousness studies (Velmans [1999]) could be rooted in the biological cell's literal separation of interior and exterior.

1.6 Substance

The set $\mathcal{X}_I$ is a partial-order category ordered by inclusion of underlying cores of things. In other words, the part-whole relationship of physical things x of y may be denoted by $x \leq y$, and so the projection of global possible-behaviors of the whole y to the local possible-behaviors of x is a surjection, denoted by $\varepsilon_I^{yx} : \eta_I^{-1}(y) \to \eta_I^{-1}(x)$, and by assumption there is a commutative diagram

$$
\begin{array}{ccc}
\eta_{I'}^{-1}y & \hookrightarrow & \eta_I^{-1}y \\
{\scriptstyle\varepsilon_{I'}^{yx}}\downarrow & & \downarrow{\scriptstyle\varepsilon_I^{yx}} \\
\eta_{I'}^{-1}x & \hookrightarrow & \eta_I^{-1}x
\end{array}
\tag{5}
$$

in which the horizontal maps are defined by restriction of possible-behaviors. A *pure substance* is a thing with core consisting of N isomorphic sub-cores whose realizations are congruent. N is the *amount* of substance. The common core and conformation is the *species* of substance. More generally, a *substance* is a union of amounts of pure substances.

1.7   Process

By "process" is meant a *thermodynamic process*, which is a possible-behavior of a thing that is a *container* of *substance*.

> In a sense, understanding the mechanism of a reaction in detail amounts to having a mental moving picture of how the atoms and electrons move as the reaction is occurring(Simons).

> The formation of new bonds decreases the number of degrees of freedom, or forms of motion, available to the atoms. That is, the atoms are less free to move in random fashion because of the formation of new bonds. The decrease in the number of molecules and the resultant decrease in motion result in fewer possible microstates and therefore a decrease in the entropy of the system (Brown et al. [2015]).

> The easiest way to consider entropy in chemistry is to associate it with all the kinds of molecular and atomic movements, referred to as degrees of freedom. The more degrees of freedom, and the more "loose" these are, the greater (and more favorable) the entropy. There are three different kinds of degrees of freedom: translational, rotational, and vibrational. Translational and rotational refer to the translation of the molecule throughout space and the tumbling of the molecule, respectively. Vibrational entropy is much more complex. Here we refer to every kind of internal motion of the molecule, such as bond stretches, bond rotations, and various forms of bond angle vibrations. The more freely that a bond rotates, a bond angle bends, or a bond stretch occurs, the more favorable the entropy. In general, the more kinds of motions and the more unconstrained those motions are, the more favorable the entropy (Anslyn and Dougherty [2006]).

If $\mathbb{S} = \|C\|$ is the space of a core, then alternatives processes $P : [a, b] \times \mathbb{S} \to \mathbb{E}$ are possible. P may just leave $\mathbb{S}$ fixed in space. Or, P may continuously rotate and translate ("tumble") $\mathbb{S}$ through space, but without change in conformation. Or, P may continuously change the conformation of $\mathbb{S}$ as it tumbles. Or, P may be composed of some previously mentioned possible types of process, such that each successive process is based on a new space obtained by loss or gain of one or more isolated joints, or struts, or of plates, between joints. If $\mathbb{S}$ is *not* connected, then each connected sub-space may separately undergo any of the aforementioned types of process, or P may be composed of processes based on the same subspaces, but possibly with struts or plates added between previously unconnected subspaces (without self-intersections). Such processes encompass all transportation and transformation of all physical things.

For biology an all-important type of process among multiple cycles of constraints is the action of enzymes, which "turn food in flesh and water into blood" (Goodsell [1996]) (Hammes [2002]) (Garcia-Viloca et al. [2004]) (Benkovic et al. [2008]).

Multiple levels of thermodynamics are directly relevant to $\mathcal{L}ife$. The basic problem of phenomenological thermodynamics is "the determination of the equilibrium state that eventually results after the removal of internal constraints" (Callen [1985]) in a closed container with barriers separating homogeneous parts "between which substances are exchanged" (Kondepudi and Prigogine [1998]). This theory is extended to a mental model of chemically reacting mixtures of pure substances within a container which is expressed by continuous-time deterministic non-linear differential equations ("mass action laws")) whose coefficients are the "reaction rate constants" (Baez and Pollard [2018]). Reaction rates can be accurately measured, and also there exist exquisite *ab initio* calculations of them (e.g., (Miller et al. [1983])). But biological organisms are not containers of continuously distributed substances in equilibrium. They are composed of relatively small

numbers of randomly fluctuating large molecules with many different shapes organized far from equilibrium in temporary stationary states. Mental models of such conditions are expressed by stochastic thermodynamics (Seifert) (Ouldridge [2018]). Stochastic chemical kinetics is based on the "chemical master equation" for the rate of change of the probability of a change in the number of molecules of each pure substance of the mixture in a container. The *propensity* for a jump from one composition to another due to a reaction is a probability per unit time, and can be calculated from a rate constant (Wolkenhauer et al. [2004]) (Lecca [2013])(Warne et al. [2019]). There is directed motion of macromolecules within cells (Scarabelli and Grant [2013]), and organisms move themselves in space (Cooper [2011]). Obviously, energy transduction is crucial for $\mathcal{L}ife$, and that occurs among molecules (Rubi et al. [2007]0) (Lipowsky et al. [2009]).

The things involved in an environment Env with an organism Org are illustrated in Fig.(7) (cf. (Mossio et al. [2016])). The chemical thermodynamic transportation and transformation constraint equations are as follows: $Fd_{Alm}^{Env} = g_0$ says that food Fd is transported from the environment Env to the alimentary tract Alm at rate g_0. $Alm_{Ntr}^{Fd} = g_1(Enz_{ALM})$ says that the transformation of food Fd into nutrients Ntr in the alimentary tract Alm is a function g_1 of the amount Enz_{Alm} of enzymes in the alimentary tract. $Ntr_{Vsc}^{Alm} = g_2(Org)$ says that the transportation of nutrients Ntr from the alimentary tract into the vasculatory system Vsc where blood Bld circulation depends via function g_2 on the Org thing. $Ntr_{V_1+\cdots+V_n}^{Vsc} = g_3(Org)$ says that transportation of nutrients from the vascular system to all the cells $V_1 + \cdots + V_n$ of the vascular system depends via g_4 on the whole organism; likewise, the transportation of nutrients from the vascular system to all the cells of the alimentary tract is represented by $Ntr_{A_1+\cdots+A_p}^{Vsc} = g_5(Org)$. In each cell A_k of the alimentary tract, transformation of nucleotides among the nutrients into RNA guided by DNA is expressed by the conjunction $\bigwedge_{1\leq k\leq p}(A_k)_{kRNA}^{Ncl} = g_5(DNA_{A_k})$; likewise, in each cell of the alimentary tract transformation of amino acids among the nutrients into enzymes is represented by the conjunction $\bigwedge_{1\leq k\leq p}(A_k)_{Enz}^{Amn} = g_6(DNA_k)$. And transformation of nucleotides into repaired DNA is expressed by $\bigwedge_{1\leq k\leq p}(A_k)_{DNA}^{Ncl} = g_7(Enz_{A_k})$. To be sure, these equations are but a caricature, and merely an allusion to the *circular* relationship between constraints and the generation, maintenance, regeneration, and ultimate decline of cells that make possible the constraints (Mossio et al. [2016]). At the end there is waste Wst. Compare Fig.(7) to the "circulatory patterns in a range of animal taxa" in (Rosslenbroich [2014], p.89).
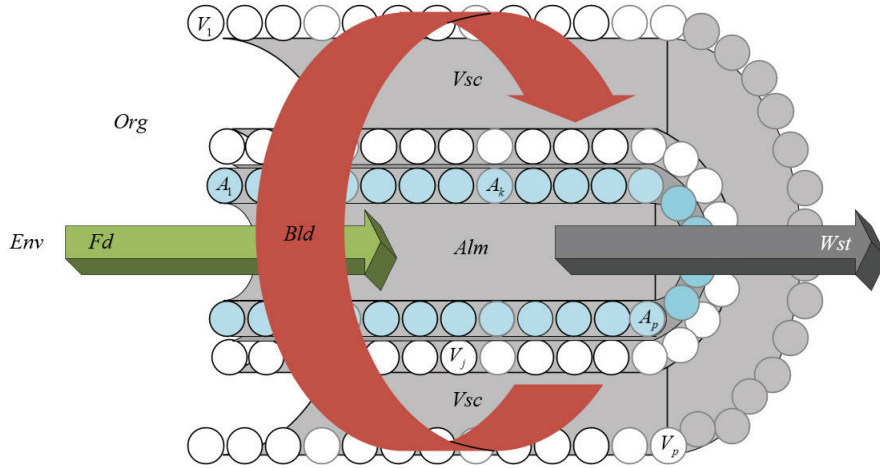

Figure 7: An environment with an organism.

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